**Homework 1**

**P1.1.2** In a one-dimensional flow of current through a semiconductor, positively-charged holes move in the positive *x*-direction at a steady rate of , and electrons move in the negative *x*-direction at a steady rate of  electrons/min. Determine the total current in mA in: (a) the positive *x*-direction, (b) the negative *x*-direction. (c) What would be the current if the holes and electrons move in the same direction? Note that the positive charge of a hole has the same magnitude as the charge of an electron, which is 1.6×10-19 C.

**Solution:** (a) The current due to positive charges moving in the positive *x*-direction adds to the current due to negative charges moving in the negative *x*-direction. The total current is: += 0.02 A ≡ 20 mA;

(b) The sign of the current is reversed to -20 mA;

(c) The currents due to holes and electrons are in opposition. The total current is: + in the direction of movement.

**P1.1.4** The charge *q* varies with time as shown in Figure P1.1.4, where *q* = 2sin(*πt*/2) C, 0 ≤ *t* ≤ 1 s. Determine the variation of the current *i* with time.

**Solution:** *i* = *dq*/*dt*, so that:

0 ≤ *t* ≤ 1 s: *i* =   A,

1 ≤ *t* ≤ 2 s: *i* = 0,

2 ≤ *t* ≤ 4 s: *i* = slope of the line = -2A,

4 ≤ *t* ≤ 6 s: *i* = slope of the line

= 1 A.

The time variation is as shown.

**P1.2.1** Given an electric field *ξ* V/m that is constant in the *x*-direction and a charge +*q* C located at the origin and free to move in the *x*-direction (Figure P1.2.1). (a) What is the magnitude and direction of the force *F* acting on *q*? (b) If *q* moves under the influence of *F* a distance *d* m, how much work is done by *F*? (c) Assuming the voltage at the origin to be zero, what is the voltage *Vd* at *x* = *d* m, bearing in mind that *ξ = -dv/dx*? (d) How is the loss in electric potential energy related to the work done by *F*? (e) Assuming the charge has a mass m kg and zero velocity at the origin, show that the KE of the charge at *x* = *d* is equal to the loss in electric potential energy.

**Solution:** (a) From basic electrostatics, *F* = *q*ξ, in the direction of the electric field, where in SI units, *q* is in coulombs, *ξ* is in volts/meter, and F is in newtons;

(b) Since *F* is constant with respect to *x*, work done by *F* is *W* = *Fd*;

(c)  V, the minus sign indicating that *v* decreases in the *x*-direction;

(d) The loss in electric PE is 0 – (-*qVd* ) = *qξd*, which is equal to *Fd*;

(e) The acceleration is *a* = *F*/*m* = *qξ*/*m* m/s2, which is constant. Hence, velocity is *at*. The distance travelled during the interval *t* is  . This gives: . The KE is , which is the loss in electric PE.

**P1.2.2** Consider the system of Figure P1.2.2, where the voltage *VAB* V between the two metal plates is maintained constant by the battery. Let there be a mechanism for moving positive charge from the lower plate to the upper plate. (a) How much work is done in moving an amount of charge +*q* C? What is the graph of *W* vs. *q*?

**Solution:** (a) In moving a charge *q* through a voltage rise *VAB*, the increase in electric PE is *qVAB*, which is equal to the work done in moving the charge.

(b) Since *W* = *qVAB*, with *VAB* constant, the graph of *W* vs. *q* is straight line of slope *VAB* passing through the origin.

**P1.3.3** The voltage drop across a certain device, and the current through it, in the direction of the voltage drop, are given by:

 *v* = sin*πt*/*2* V, *i* = cos*πt*/*2* A

(Figure P1.3.3). (a) Determine whether the device absorbs or delivers power during each of the quarter cycles of one period. (b) Derive the power at any instant as a function of time. How do you interpret positive values and negative values of power? What is the maximum magnitude of instantaneous power? What is the average power over a 4-s period?

**Solution:** (a) Power is absorbed during quarter cycles having the same sign and is delivered during quarter cycles of opposite sign. Thus, power is absorbed during the intervals 0 ≤ *t* ≤ 1 and 2 ≤ *t* ≤ 3, and power is delivered during the intervals 1 ≤ *t* ≤ 2 and 3 ≤ *t* ≤ 4;

(b) *p* = *vi* = sin(*πt*/2)cos(*πt*/*2*) W = 0.5sin*πt* W; *p* > 0 is power absorbed, *p* < 0 is power delivered; the maximum magnitude of the instantaneous power is 0.5 W, the amplitude of 0.5sin*πt*, and the average is zero.

**P1.3.5** The voltage drop across a certain device, and the current through it, in the direction of the voltage drop, are shown in Figure P1.3.5. Determine: (a) the charge *q* through the device at the end of each 1 s interval from  to  (b) the instantaneous power  during the aforementioned intervals; and (c) the total energy consumed by the device.

**Solution:** (a) 0 < *t* < 1: *q* = ,

*t* = 2: *q* =  

*=* 2.5 mC, which is the area of a trapezoid of width 1 s and average height 2.5 mA,

*t* = 3: *q* = 2.5 +2.5 +5.5 mC, added area of rectangle 3 mA×1 s,

*t* = 4: *q* = 5.5 +5.5 +8.5 mC, added area of rectangle 3 mA×1 s,

*t* = 5: *q =* 8.5 +8.5 +8.5 += 11 mC, added area of trapezoid of width 1 s and average height 2.5 mA,

*t* = 6: *q* = 11 + 11 mC;

(b) ;

mW,

mW,

mW,

mW, ;

(c) + +=  = 136/3 = 45.3 mJ.

**P1.3.6** The voltage drop across a certain device, and the current through it, in the direction of the voltage drop, are given by:

   

 *v* and *i* are zero elsewhere

Sketch the variation of *v* and *i* with time. (a) At what instant of time is maximum instantaneous power absorbed by the device? (b) At what instant is it zero? (c) What is the energy delivered to the element at *t* = 2 s and at  (d) Over what time interval does the device absorb power, and over what time interval does it deliver power?

**Solution:** *p = vi* = (2*t* + 1)(4 – 2*t*) = .

(a) ; *p*maxs;

(b) *p* = 0 ; *p* = 0 when either *v* or *i* is zero;

(c) At *t* = 2s: *w* = 9.3 mJ,

at *t* = 4s: *w* = -21.3 mJ;

(d) power is absorbed by device for *t* < 2 s, and is delivered by device for

2 < *t* < 4 s.

**P1.3.7** The voltage drop  across a certain device, and the current  through it, in the direction of the voltage drop, are related by:

  

   and

(a) Determine the power absorbed by the load when *v* = 1 V, and when *v* = 2 V; (b) at what value of *v* is the instantaneous power a maximum?

(c) If V,  what is the total charge that passes through the device from *t* = 0 to ?

**Solution:** (a) ; *p = vi* = *v*(8 – 2*v*2) = –2*v*3 + 8*v*, and *p* = 0, ,

at *v* = 1 V; *p* = 6 W; at *v* = 2 V, *p* = 0;

(b) , ; *pmax* V;

(c) ;C.